However, in terms of D, the required h_a is $2.4D$ in all four cases. For a given δ, doubling D will approximately double both the absolute values of Lu and htp. In turn, this will double the required value of h_a, but the ratio h_a/D will remain the same. The main effect of δ lies in its second-order influence on Lu.

h_a will also depend on the protuberance shape. For fixed flow conditions, experiments at Princeton show that Lu for a flat faced fin of thickness t is approximately twice that for a hemi-cylindrically blunted fin with $D=t$. Since ϕ stays the same, both htp and the required h_a for the former are about twice those of the latter. For a given value of δ, h_a/δ will differ by a factor of 2 for the two cases, whereas h_a/htp will be about the same.

The ideas above explain several apparent anomalies existing in the literature. For example, Waltrup et al.,[15] observed the asymptotic result at an h/δ of about 10, whereas Price and Stallings,[3] who tested five sweptback fin models in the range $0.17 \leq h/\delta \leq 1.07$ found that it had already occurred at the lowest value of h/δ. Waltrup's flow conditions input to Eqs. (1) and (2) gave $htp = 1.9$ cm (or 4.3δ), suggesting that the asymptotic result would occur at 2-3 times this height (i.e., around 9-13δ), which was the case. For Price and Stallings' flow, the equations give $htp \simeq 0.07\delta$ so that it was not surprising that even at an h/δ of 0.17 the asymptotic result had already occurred.

In the fully laminar case, recent measurements by Hung and Clauss[13] show that Lu also scales with D, and typically is between 9 and $12D$. In addition, the exponents of M and Re and the constant K in Truitt's relation [i.e., Eq. (2)] will differ from those for turbulent flow.

Concluding Remarks

The ratio h/δ is not the physically relevant parameter when determining if a given protuberance will generate the asymptotic flowfield. Physically, the latter occurs when the fin height h is of order 2-3 htp and, since for a given flow htp depends primarily on D, the important parameter is h/D. A unique value of h/D, valid for arbitrary conditions, cannot be specified since htp depends also on M and Re.

Acknowledgments

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Supercritical Swirling Flows in Convergent Nozzles

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Introduction

It has generally been assumed that the topic discussed in this Note has already been more than adequately covered in numerous papers (fairly comprehensive lists of references are given in Refs. 1 and 2). Previous authors, however, have apparently overlooked the fact that the quasicylindrical theory for compressible swirling flow in convergent-divergent nozzles leads to a paradox when applied to convergent nozzles. Accordingly, the present Note sets out to describe this paradox and to present a simple theory for supercritical swirling flow in convergent nozzles. Some results for mass flux and thrust coefficients are also given.

For nonswirling supercritical flows it is generally assumed that one-dimensional theory is equally applicable to convergent and convergent-divergent nozzles. The theory is found to give good results when the area change is gradual and the nozzle wall curvature small. It has also been generally accepted, at least implicitly, that the quasicylindrical theory for swirling compressible flow is equally applicable to both types of nozzle. There has been considerable difficulty, though, in defining a suitable choking criterion for swirling nozzle flows. Many previous investigators[3-5] have used the concept of maximum-mass-flux to obtain a choking criterion. Norton et al.[6] and Carpenter and Johannesen,[1,2] however, made no additional assumptions about choking beyond those usually made for quasicylindrical theory. The latter showed that the maximum-mass-flux criterion is not strictly valid but gave reasonable results for most swirling flows in convergent-divergent nozzles. They also showed that, owing to the

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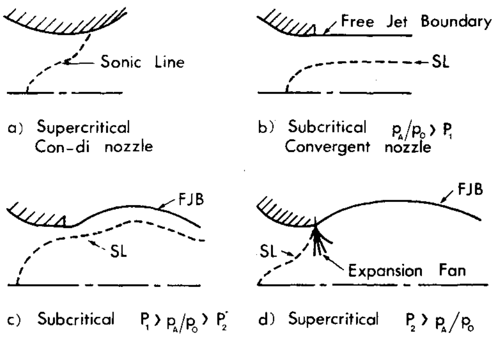

Fig. 1 Sonic lines and free jet boundaries for swirling nozzle flows.

variation of axial velocity across a nozzle section, the sonic line is curved for swirling flow even when the effects of throat curvature are neglected (see Fig. 1a). For all the types of swirling flow investigated, the sonic line intersects the axis upstream and the nozzle wall downstream of the throat. As explained below, this would imply that quasicylindrical theory cannot be applied to convergent nozzles.

For axisymmetric swirling flows the sonic line is defined with reference only to the velocity component in the meridian plane; i.e., to the axial velocity for quasicylindrical theory. Thus, it is quite possible for the flow to be locally supersonic (based on the total velocity magnitude) but at the same time subcritical. This situation arises because the projection onto the meridian plane of the Mach cone associated with the total velocity vector would extend some way upstream if the velocity component in the meridian plane were subsonic. This would allow small disturbances to propagate upstream following spiral paths. Consequently, in order to support a pressure difference between the exit of a convergent nozzle and ambient conditions, for quasicylindrical flow the axial velocity at the nozzle lip must be greater than or equal to the local speed of sound. This requirement is incompatible with the sonic-line shape predicted by quasicylindrical theory at the throat of a convergent-divergent nozzle.

Having established that quasicylindrical theory cannot be applied to supercritical convergent-nozzle flows the question arises as to what sort of flow does exist in the hypothetical case of a convergent nozzle with very gradual area change. There is no problem in applying quasicylindrical theory to subcritical flows where the back pressure ratio, p_A/p_0, (where p_A and p_0 are ambient and stagnation pressure, respectively) is greater than or equal to P_1 (the ratio of the pressure at the throat wall of a choked convergent-divergent nozzle to stagnation pressure). This case is illustrated in Fig. 1b and dealt with in Ref. 7. When p_A/p_0 lies between P_1 and P_2 (the pressure ratio at which the axial velocity at the nozzle lip just equals the local speed of sound) the nozzle flow is still subcritical. However, there must be some mechanism by which the flow may reach higher velocities at nozzle exit than would exist in the throat of a convergent-divergent nozzle. It is suggested here that the free jet boundary now has finite curvature at nozzle exit. The curvature would need to be sufficiently large to increase the velocity at exit so that no pressure difference exists between exit and ambient at the nozzle lip. This case is illustrated in Fig. 1c. Finally, when $p_A/p_0 < P_2$ the nozzle flow becomes supercritical. A pressure difference can now exist at the nozzle lip. The slope of the free jet boundary is no longer equal to that of the wall streamline immediately upstream of the nozzle exit. Again the resulting streamline curvature is presumed to be sufficient to ensure that the sonic line intersects the nozzle lip. This case is illustrated in Fig. 1d.

Although quasicylindrical theory is no longer strictly applicable to supercritical swirling flows in convergent nozzles, a simple approximate theory can still be developed. This theory is described below and follows a similar procedure to that of Ref. 7 with the additional assumption that the axial velocity equals the local speed of sound at the nozzle lip.

Analysis

It is assumed that the swirl is produced by fixed vanes. This implies that for an inviscid non-heat-conducting gas the entropy and stagnation enthalpy are uniformly constant throughout the flow. Under these conditions it was shown in Ref. 7 that for quasicylindrical theory the axial velocity, w, at nozzle exit is given by

$$w = \left[w_{\text{ex}}^2 + 2\int_r^{R_{\text{ex}}} \frac{v}{r'} \frac{\mathrm{d}(r'v)}{\mathrm{d}r'} \mathrm{d}r' \right]^{1/2} \tag{1}$$

where v is the swirl velocity at nozzle exit, subscript ex indicates conditions at the nozzle lip, r is the radial coordinate and R_{ex} is the radius of the exit section.

The pressure p, density ρ, and local speed of sound a, at the nozzle exit are obtained from the isentropic flow relations, viz.

$$\left[\frac{a}{a_0}\right]^2 = \left[\frac{\rho}{\rho_0}\right]^{\gamma-1} = \left[\frac{p}{p_0}\right]^{(\gamma-1)/\gamma} = 1 - \frac{\gamma-1}{\gamma+1}(W^2+V^2) \tag{2}$$

where $W = w/a_*$, $V = v/a_*$, a_* is the critical speed of sound, γ is the ratio of the specific heats and subscript 0 denotes stagnation conditions.

As suggested above, w_{ex} can be determined by setting it equal to the local speed of sound, viz.

$$w_{\text{ex}} = a_{\text{ex}} \tag{3}$$

When use is made of Eq. (2), Eq. (3) gives

$$W_{\text{ex}} = \left\{ 1 - \frac{\gamma-1}{\gamma+1} V_{\text{ex}}^2 \right\}^{1/2} \tag{4}$$

The quantities of most practical significance as regards nozzle performance are the mass flux, impulse function and thrust. Nondimensional coefficients for these three quantities can be defined as follows:

$$C_m = 2\int_0^{R_{\text{ex}}} \rho w r \mathrm{d}r / \rho_* a_* R_{\text{ex}}^2$$

$$C_{nx} = 2\int_0^{R_{\text{ex}}} (p+\rho w^2) r \mathrm{d}r / (p_* + \rho_* a_*^2) R_{\text{ex}}^2$$

$$C_t = t/\rho_* a_*{}^2 \pi R_{\text{ex}}^2 \tag{5}$$

where ρ_* and p_* are the density and pressure corresponding to a_* and t is the thrust. The integrals are evaluated at the nozzle exit.

The thrust generated by a nozzle, attached to a turbojet moving at flight speed V_F is given to good approximation by

$$t = 2\pi\int_0^{R_{\text{ex}}} (p+\rho w^2) r \mathrm{d}r - 2\pi V_F \int_0^{R_{\text{ex}}} \rho w r \mathrm{d}r - \pi p_A R_{\text{ex}}^2 \tag{6}$$

Using Eq. (6) together with Eqs. (5) leads to

$$C_t = \frac{\gamma+1}{\gamma} C_{nx} - \frac{V_F}{a_*} C_m - \frac{1}{\gamma}\left[\frac{p_A}{p_0}\right]\left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)} \tag{7}$$

If the swirl profile at nozzle exit is given, the axial velocity can be determined from Eqs. (1) and (4). The density and pressure distributions can then be determined from Eq. (2). With the axial velocity, density and pressure distributions determined the coefficients C_m and C_{nx} can be evaluated by numerical quadrature.

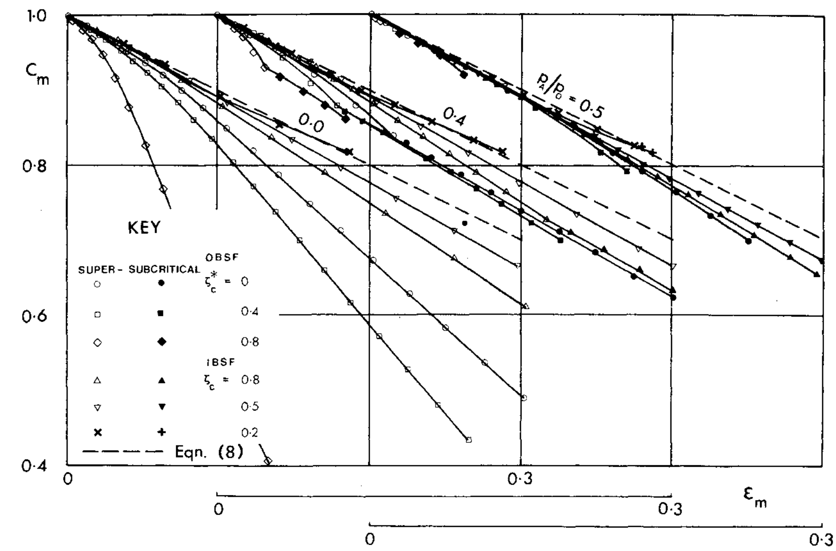

Fig. 2 Variation of mass-flux coefficient with swirl intensity ($\gamma = 1.4$).

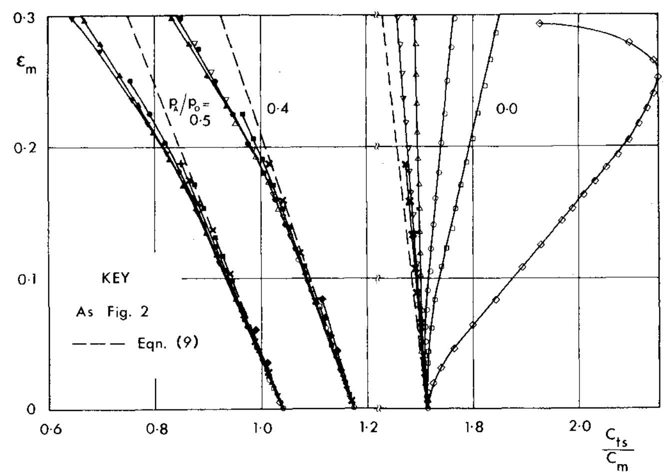

Fig. 3 Variation of static specific thrust coefficient with swirl intensity ($\gamma = 1.4$).

In the case of weak swirl analytical expressions can be obtained for C_m and C_{nx} by following a procedure similar to that described in Refs. 1 and 2. Accordingly, the right-hand side of Eq. (1), with Eq. (4) substituted for W_{ex}, is expanded and terms of $0(V^4{}_{ex})$ neglected. Approximate expressions for ρ/ρ_0 and p/p_0 are obtained from Eq. (3), with use of Eqs. (1) and (4), by expanding and neglecting terms of $0(V^4{}_{ex})$. These approximate expressions for W, ρ/ρ_0 and p/p_0 are substituted into the integrands in Eqs. (5) and the integrations carried out again neglecting terms of $0(V^4{}_{ex})$, to obtain the following approximate expressions:

$$C_m = 1 - \epsilon_m, \qquad C_{nx} = 1 - \frac{2\gamma}{\gamma+1}\epsilon_m \tag{8}$$

where

$$\epsilon_m = \int_0^1 V^2 \zeta \mathrm{d}\zeta \text{ and } \zeta = r/R_{ex}$$

An approximate analytical expression for the thrust coefficient can be obtained by using Eqs. (8) with Eq. (7). For example, the coefficient of specific static thrust is given by

$$\frac{C_{ts}}{C_m} = \left\{\frac{\gamma+1}{\gamma} - \frac{p_A}{\gamma p_0}\left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)}\right\}(1+\epsilon_m) - 2\epsilon_m \tag{9}$$

Results and Discussion

Accurate values of C_m and C_{ts}/C_m have been computed for two families of swirl distributions. The first of these is defined as follows:

$$V = \Omega\zeta : 0 \le \zeta \le \zeta_c; \qquad V = \Omega\zeta_c^2/\zeta : \zeta_c \le \zeta \le 1 \tag{10}$$

This represents an "inner-biased"[2] swirling flow (IBSF) with a core, of radius $\zeta_c R_{ex}$, which is in solid-body rotation; outside the core there is free-vortex flow. The parameters Ω and ζ_c can both be varied. The second family is defined as follows:

$$V = 0 : 0 \le \zeta \le \zeta_c^*; \qquad V = \frac{\zeta - \zeta_c^*}{1 - \zeta_c^*} V_{ex} : \zeta_c^* \le \zeta \le 1 \tag{11}$$

This represents an "outer-biased"[2] swirling flow (OBSF) with a nonswirling core of radius $\zeta_c^* R_{ex}$, surrounded by an annulus of swirling flow. Again, two parameters, V_{ex} and $\zeta^*{}_c$, can be varied. Both types of flow were investigated experimentally by Whitfield[8] in supercritical convergent nozzles.

In Figs. 2 and 3 the calculated values of C_m and C_{ts}/C_m, corresponding to various types of OBSF and IBSF, are plotted against the swirl parameter, ϵ_m, for particular back-pressure ratios. The variations of C_m and C_{ts}/C_m with ϵ_m according to Eqs. (8) and (9) respectively are also plotted. It will be seen from Figs. 2 and 3 that, for a fixed nonzero back-pressure ratio, the flow becomes subcritical at sufficiently high-swirl intensities. For a given type of swirling flow the calculated point which is plotted as the supercritical point corresponding to the highest swirl intensity is, in fact, the critical point. It can be seen that Eqs. (8) and (9) appear to be reasonable approximations for moderate swirl intensities providing the back-pressure ratio is not too low. The subcritical points were calculated following the method described in Ref. 7. Note that for a given nonzero value of p_A/p_0 the subcritical points virtually collapse onto single curves even at high swirl levels. This confirms the conclusion made in Ref. 7 that ϵ_m may be regarded as a universal swirl parameter for subcritical swirling flows.

The supercritical values of C_m and $\{(\gamma+1)/\gamma\}C_{nx}/C_m$ can be obtained from the results corresponding to $p_A/p_0 = 0.0$ in Figs. 2 and 3. Consequently, the mass flux and thrust for supercritical swirling flows can be calculated for any value of p_A/p_0 by making use of Eq. (7) and the results plotted in Figs. 2 and 3. The present theoretical calculations are not compared to Whitfield's[8] experimental data since the available information was insufficient for a meaningful comparison.

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Flow Nonuniformity in Low Pressure Shock Tubes under Nonasymptotic Conditions

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Nomenclature

ℓ = distance behind shock front
M = shock Mach number
t = time
u = velocity
ρ = density

Subscripts

m = conditions at maximum test time
1 = conditions ahead of advancing shock wave
2 = conditions in shock heated gas
s = conditions at shock front

Introduction

FLOW nonuniformities associated with the presence and growth of boundary layers behind incident shock waves continue to attract considerable experimental and theoretical attention especially in the context of studying basic collisional and radiative processes in shock heated or shock-generated plasmas. This follows since variations in such fluid properties as temperature and density in the shock heated gas, induced by these nonuniformities, must be known accurately to allow reliable kinetic data to be obtained.[1] Fortunately, an elegant (and tractable) theory has been formulated by Mirels[1,2] and has already been subjected to experimental verification.[3] However, these previously reported experimental studies were restricted to a single gas, argon, and also were concerned with fully developed flow conditions in which a limiting separation, ℓ_m, between the shock front and contact surface, had been established.

However, in many cases of interest, it may be necessary to operate under partially developed flow conditions as a result of constraints in the overall length or diameter of the flow channel itself or the desired range of operating pressures. In such partially developed flows the need to correct for flow nonuniformities associated with the boundary-layer growth can also be anticipated. In the work reported here, experimental data for a range of gases and gas mixtures are presented which confirm the validity of the Mirels treatment in this case. A brief outline of the background theory, experimental approach, and results is presented below.

Theory

Mirels[1,2] has shown that flow nonuniformities associated with laminar boundary-layer growth in medium and strong shock waves can be derived starting with the mass conservation relationship

$$(\rho_2 U_2/\rho_{2s} U_{2s}) \equiv (U_2/U_{2s}) = (1-(\ell/\ell_m)^{1/2}) \tag{1}$$

In this expression ρ and U denote density and shock velocity, respectively, in shock-front fixed coordinates. The subscript 2 refers to the shock heated gas and the additional subscript, s, refers to the conditions at the shock front. The distance behind the shock front is denoted by ℓ and in a fully developed flow ℓ_m defines the location of the contact surface. Fox et al.[3] integrated Eq.(1) to obtain an expression for t_p the time required for a particle to travel from the shock front to any station ℓ in the shock heated gas. When combined with the expression $t_L(\equiv \ell/U_{2s})$ representing the time (as measured in the laboratory frame of reference) for the shock to move a distance ℓ the following relationship is obtained[3]:

$$(t_p/t_L) = -2(\ell/\ell_m)^{-1}[(\ell/\ell_m)^{1/2} + \ln[1-(\ell/\ell_m)^{1/2}] \tag{2}$$

Valid for laminar boundary layers and strong shocks, Eq.(2) must be applied (in addition to the normal density-ratio factor) to convert laboratory to particle time scales. Although Eq. (2) was initially derived and subsequently experimentally confirmed in the context of fully developed flow regimes,[3] Mirels has indicated that it should be equally valid in partially developed regimes provided shock attenuation effects remain small.[2] A primary objective of the work reported here was to test the validity of this hypothesis. As discussed below this was achieved by comparing particle paths in the shock heated gas with those predicted using Eq. (2).

Experimental

A 4 cm i.d. pressure-driven shock tube of similar design and construction to that described previously by Cunningham and Hobson[4] was used in this study. Spectroscopically pure rare gas samples or gas mixtures buffered with rare gases were introduced into the evacuated-flow tube to a pressure between 1 to 12 Torr. Shock waves in the range Mach 1.6 to Mach 6.0 were transmitted into the low-pressure test gas by rupturing a thin diaphragm.[4] Under these conditions, the shock heated gas was nonionized and had a translational temperature between 500 and 3500 K. Also, at the concentration levels (less than 1%) used, the presence of molecular additives such as N_2, O_2, or H_2O were found not to

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